Spinning Vehicle Nutation Damper

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The linear acceleration at a point away from the center of mass on a spinning body may be used as the driving force for a damper system. The spinning vehicle equations of motion, including the motion of a sliding mass, are derived by the methods of classical mechanics. Solutions of the resulting fourth-order differential equations for the angular rates are discussed, and the numerical results are presented in graphical form for a variety of parameter values. Approximate design formulas are given, and the test program used to verify the damper action is discussed and illustrated.

Nomenclature

a	= distance along the axis of symmetry from c.g. to neutral position of the slider, in.
c	= damping coefficient, lb-sec/in.
c_c	= critical damping coefficient, lb-sec/in.
$F/2$	= damping constant, 1/sec
F_f	= viscous friction force, lb
F	= Rayleigh's dissipation function, lb-in./sec
F_s	= spring force, lb
g	= acceleration of gravity, in./sec^2
I_x, I_y, I_z	= principal moments of inertia
$\bar{i}, \bar{j}, \bar{k}$	= unit vectors in a rotating reference frame
i	= imaginary number $(-1)^{1/2}$
K	= spring constant, lb/in.
$\bar{L}$	= angular momentum, in.-lb-sec
L	= Lagrangian function, lb-in.
m	= mass of the sliding weight, lb-sec^2/in.
N_i	= applied moment about the ith-axis, lb-in.
p, q, r	= spin, pitch, and yaw rates, respectively, rad/sec
q_i	= generalized coordinate
s	= Laplace transform operator
T	= kinetic energy, lb-in.
t	= time, sec
$\hat{t}$	= time to damp oscillations to 10% of their initial value, sec
V	= potential energy, lb-in.
$\bar{v}$	= velocity vector in Newtonian reference frame
x, y, z	= Cartesian coordinates of rotating system
X, Y, Z	= Cartesian coordinates of inertially fixed system
ϕ, θ, ψ	= spin, pitch, and yaw angles, respectively, rad
λ	= damping ratio c/c_c
$\bar{\rho}$	= position vector in rotating coordinate system
Δ	= characteristic function of Laplace transform solution (see text)
σ	= moment of inertia ratio, I_x/I_y
Ω	= angular rate of spinning system, $p(\sigma - 1)$(rad/sec); $\sigma > 1$
δ	= spin axis angular displacement
$\bar{\omega}$	= angular velocity vector
ω_n	= undamped natural frequency of the sliding mass-spring
$(\cdot)$	= differentiation with respect to time
$(\ )\cdot(\ )$	= scalar multiplication
$(\ )\mathbf{X}(\ )$	= vector product

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Introduction

STUDIES of the separation dynamics of spin-stabilized satellites and probes indicate the possibility of unbalanced moment disturbances during the separation process that result in excessively large nutation or coning motions of the body in its free flight. This motion causes detrimental fluctuations of the antenna or sensor alignment. The complexity and expense of adding an active control system to remove this coning leads to the consideration of passive devices. By nature, these devices are mechanically simple; however, their analysis and subsequent design are often complex. Typical examples include sliding masses and liquid filled toruses.

Such devices have been under discussion and experimentation recently.[1] This paper presents design information on a simple sliding mass-spring passive damping system as a contribution to the growing pool of information on these devices. This particular device can be excited into a form of resonance which causes a high energy removal rate with relatively small added weight to the vehicle.

For a spinning nutating body with the spin moment of inertia larger than the lateral moments of inertia, the removal of energy from the system is dynamically stabilizing and will cause a decrease in the amplitude of nutation. However, determining the time of amplitude decrease to some desired value is not always straightforward.

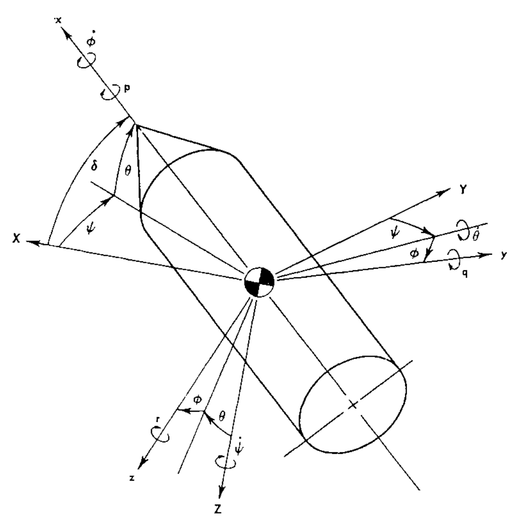

Fig. 1 Orientation of x, y, z body axes relative to X, Y, Z inertial axes. The relationship is described by Euler angles ψ, θ, ϕ. The positive sense of angular rates about body axes is shown as p, q, r.

The general requirements for a nutation damper may be stated as follows: 1) the damper must reduce the body nutation angular rates (lateral only) and angular amplitudes to some minimum value in a specified time; 2) the damper must not appreciably disturb the motion of the parent or launch vehicle prior to separation of the satellite when the moment of inertia ratio may be unfavorable; 3) the damper must lend itself to a reasonably accurate method of analysis; and 4) the damper must lend itself to evaluation and verification by a fairly simple test.

A device that satisfies all of the forementioned requirements is a relatively small mass-spring arrangment wherein the mass can be excited into large amplitude motion on a confined shaft or guide. This paper presents a method of analysis for a system and the results of a simple test program demonstrating the action.

Equations of Motion

Consider a body with a fixed center of gravity (no translational accelerations). Noting Fig. 1, the Euler equations of motion may be written[2,3] as follows:

$$I_x\dot{p} - qr(I_y - I_z) = N_x$$
$$I_y\dot{q} - rp(I_z - I_x) = N_y$$
$$I_z\dot{r} - pq(I_x - I_y) = N_z$$

If the x axis is the axis of symmetry of the body, and the following definitions are made,

$$I_y = I_z$$
$$\Omega = p(I_x - I_y)/I_y$$

the following set of equations is obtained:

$$\dot{p} = N_x/I_x$$
$$\dot{q} + \Omega r = N_y/I_y$$
$$\dot{r} - \Omega q = N_z/I_y$$

The origin of the i-j-k triad is fixed at the center of gravity of the body and rotates with the body (Fig. 2). The housing of the sliding mass spring is rigidly fixed in the body and oriented parallel to the y axis. The only applied moments to the body are those resulting from the motion of the sliding mass. The forces producing the moments are the spring force and the viscous friction force. Denoting ω_n as the undamped natural frequency of the sliding mass spring and λ as the damping ratio, i.e.,

$$\omega_n = (K/m)^{1/2}$$
$$\lambda = c/c_c = c(Km)^{1/2}/2$$

the viscous friction force may be written as

$$F_f = 2\lambda\omega_n\, m\dot{y} \text{ (for harmonic motion)}$$

and the spring force and applied moments to the body are

$$F_s = K_y$$
$$N_x = N_y = 0$$
$$N_z = 2\lambda\omega_n ma\dot{y} + Kay$$

where a is the distance along the x axis from the center of mass of the body to the neutral position of the sliding mass spring (Fig. 2).

The three Euler equations become

$$\dot{p} = 0$$
$$\dot{q} + \Omega r = 0$$
$$\dot{r} - \Omega q - (2\lambda\omega_n ma/I_y)\dot{y} - (Ka/I_y)y = 0$$

Next, it is required that one more independent equation describing the motion of the sliding mass coupled with the main body be developed. Lagrangian methods afford a means of doing this and including the effect of viscous damping. The form of Lagrange's equation that is applicable is[3]

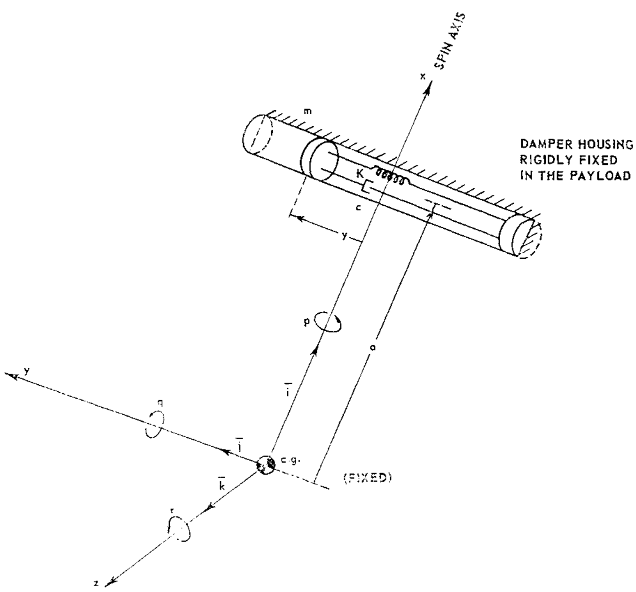

Fig. 2 Damper configuration.

$$(d/dt)(\partial L/\partial \dot{q}_i) - (\partial L/\partial q_i) + (\partial F/\partial \dot{q}_i) = 0$$

where $L = T - V$, Langrangian function; T = kinetic energy; V = potential energy; and F = Rayleigh's dissipation function. Neglecting the rotational kinetic energy of the sliding mass in comparison to the rotational kinetic energy of the body, and noting that the center of mass of the body is fixed, the terms are

$$T = (\tfrac{1}{2})m(\bar{v}\cdot\bar{v}) + (\tfrac{1}{2})I_x p^2 + (\tfrac{1}{2})I_y q^2 + (\tfrac{1}{2})I_y r^2$$

where

$\bar{v}$ = the velocity of the sliding mass in inertial space

$V = (\tfrac{1}{2})Ky^2$

$F = (\tfrac{1}{2})(2\lambda m\omega_n)\dot{y}^2$

Now

$$\bar{v} = (d/dt)(\bar{\rho}) + \bar{\omega} \times \bar{\rho}$$
$$\bar{\rho} = a\bar{\imath} + y\bar{\jmath}$$
$$\bar{\omega} = p\bar{\imath} + q\bar{\jmath} + r\bar{k}$$

The Lagrangian function becomes

$$L = (m/2)(r^2y^2 + \dot{y}^2 + 2ar\dot{y} + a^2r^2 + p^2y^2 - 2a\,pqy + q^2a^2) + (I_x/2)p^2 + (I_y/2)q^2 + (I_y/2)r^2 - (K/2)y^2$$

Note that $I_z = I_y$. The equation of motion for the y coordinate is

$$\ddot{y} + 2\lambda\omega_n\dot{y} - (r^2 + p^2 - \omega_n^2)y + apq + a\dot{r} = 0$$

Finally, the set of equations which defines the motion of the body-sliding mass system is

$$\dot{p} = 0$$
$$\dot{q} + \Omega r = 0$$
$$\dot{r} - \Omega q - (2\lambda\omega_n ma/I_y)\dot{y} - (Ka/I_y)y = 0$$
$$\ddot{y} + 2\lambda\omega_n\dot{y} - (r^2 + p^2 - \omega_n^2)y + apq + a\dot{r} = 0$$

To obtain the general analytical solution of the set, Laplace transform methods may be employed if the equations are

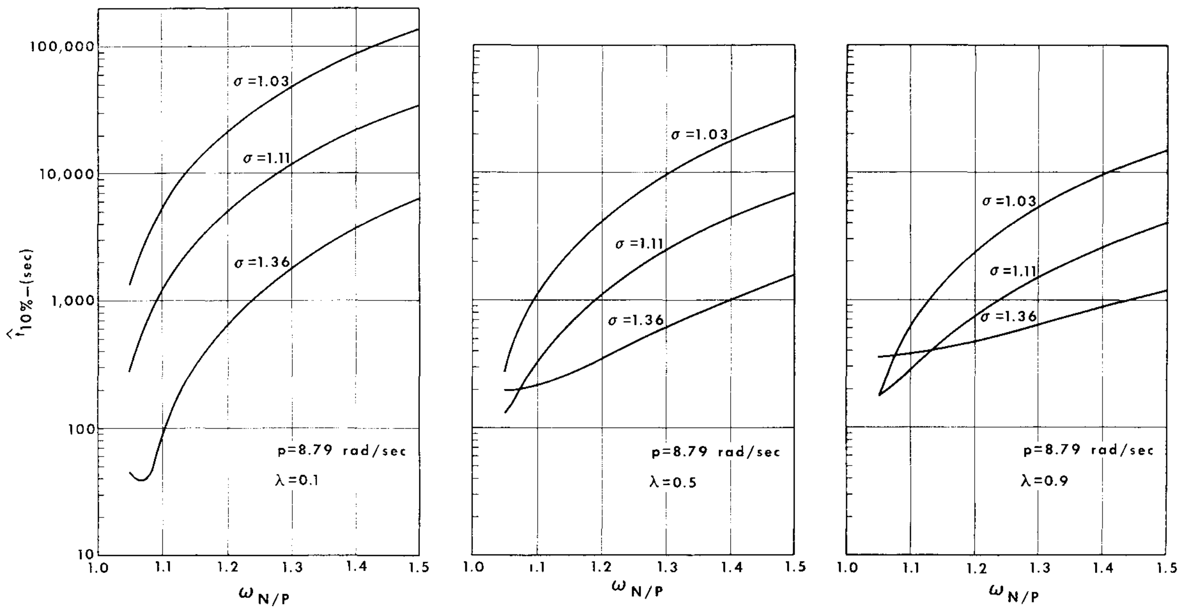

Fig. 3 Computer solution for damping times: $ma^2/I_y = 6.46 \times 10^{-4}$; $p = 8.79$ rad/sec.

linearized by neglecting r^2, which is small in comparison to p^2. For simplicity, consider the only nonzero initial condition to be

$$r(o) = r_0$$

Denoting the Laplace transform of x as x^* and transforming the linearized equations

$$sq^* + \Omega r^* = 0$$

$$-\Omega q^* + sr^* - [(2\lambda m\omega_n a/Iy)s + Ka/I_y]y^* = r_0$$

$$apq^* + asr^* + [s^2 + 2\lambda\omega_n s + (\omega_n{}^2 - p^2)]y^* = ar_0$$

Using determinant methods, the preceding equations may be solved for the Laplace transforms of the variables q, r, y. The "characteristic function" of the system is obtained from the determinant of the coefficients of the Laplace transforms:

$$\Delta = s^4 + 2\lambda\omega_n[1 + ma^2/I_y]s^3 + [\omega_n{}^2 - p^2 + \Omega^2 + Ka^2/I_y]s^2 + 2\lambda\omega_n[\Omega^2 - \Omega pma^2/I_y]s + \Omega^2[\omega_n{}^2 - p^2 - pKa^2/\Omega I_y]$$

The Laplace transforms, for the given initial condition, are

$$q^*/r_0 = -\Omega\{s^2 + 2\lambda\omega_n[1 + ma^2/I_y]s + \omega_n{}^2 - p^2 + Ka^2/I_y\}/\Delta$$

$$r^*/r_0 = s\{s^2 + 2\lambda\ \omega_n[1 + ma^2/I_y]s + \omega_n{}^2 - p^2 + Ka^2/I_y\}/\Delta$$

$$y^*/r_0 = a[\Omega^2 + p\Omega]/\Delta$$

To determine the response of the system, the inverse Laplace transforms of the forementioned equations must be found. This involves finding the roots of the characteristic function. The roots were obtained for various combinations of parameters using a digital computer program. For all of the cases studied, two of the roots occurred as a complex conjugate pair with negative real parts[4] and an imaginary component near the body rotational frequency Ω, as viewed from the body axis system. This agrees with the physical requirement that the damper have only a small effect on the system natural frequency and that the ensuing body motion be a damped harmonic motion. The other pair of roots occurred as either unrepeated negative real roots (damped motion of the slider) or as a complex conjugate pair with negative real parts (damped harmonic motion of the slider) for $\omega_n > p$. The case of $\omega_n < p$ represents an unstable situation where the slider moves out to the end of the case and stays there.

The time $\hat{t}$ required to damp the q or r rates to 10% of their initial value is determined by the real part of the first pair of roots discussed. The roots are of the form $\alpha \pm i\beta$, $\beta \approx \Omega$, and $\hat{t} = 2.3/|\alpha|$. The results are plotted as $\hat{t}$ vs ω_n/p in Figs. 3–5. The curves show the critical dependance of the damping time on the ratio ω_n/p for a given slider mass, the least time occurring when ω_n is slightly larger than p.

The maximum amplitude y_{max} of the slider motion is, of course, an important design consideration and can be determined from the transient solution for y^*/r_0. The initial lateral rate r_0 must be known as well as the other parameters of the system. An approximate solution for y_{max} is given in the next section.

Approximate Formulas

On the assumption that the sinusoidal character of the spinning body oscillatory motion is not greatly affected, an approximate formula can be derived for the damping factor of the form

$$r = r_0 \exp(-Ft/2) \cos\Omega t$$

With the damper mounted perpendicular to the spin axis and offset a distance a along the spin axis,

$$F/2 = ma^2\lambda\omega_n\sigma(\sigma - 1)[1 + (\sigma - 1)^2]/I_y\{[(\omega_n/p)^2 - 1 - (\sigma - 1)^2]^2 + 4\lambda^2(\omega_n/p)^2(\sigma - 1)^2\}$$

where σ is the moment of inertia ratio.

This equation is derived by determining the transform of y^*/r^* from the equation for y^*

$$\frac{y^*}{r^*} = \frac{\sigma ap\Omega}{s[s^2 + 2\lambda\omega_n s + (\omega_n{}^2 - p^2)]}$$

This is substituted into the second equation of motion for the system

$$-\Omega q^* + sr^* - \left[\frac{2\lambda m\omega_n as}{I_y} + \frac{ka}{I_y}\right] \times \frac{(\sigma\ ap\ \Omega)r^*}{s[s^2 + 2\lambda\omega_n s + (\omega_n{}^2 - p^2)]} = r_0$$

The assumption that the system excitation is a harmonic function means that, initially, the motion is much the same as if $s = i\Omega$. The resemblance to the equation

$$-\Omega q^* + sr^* + Fr^* = r_0$$

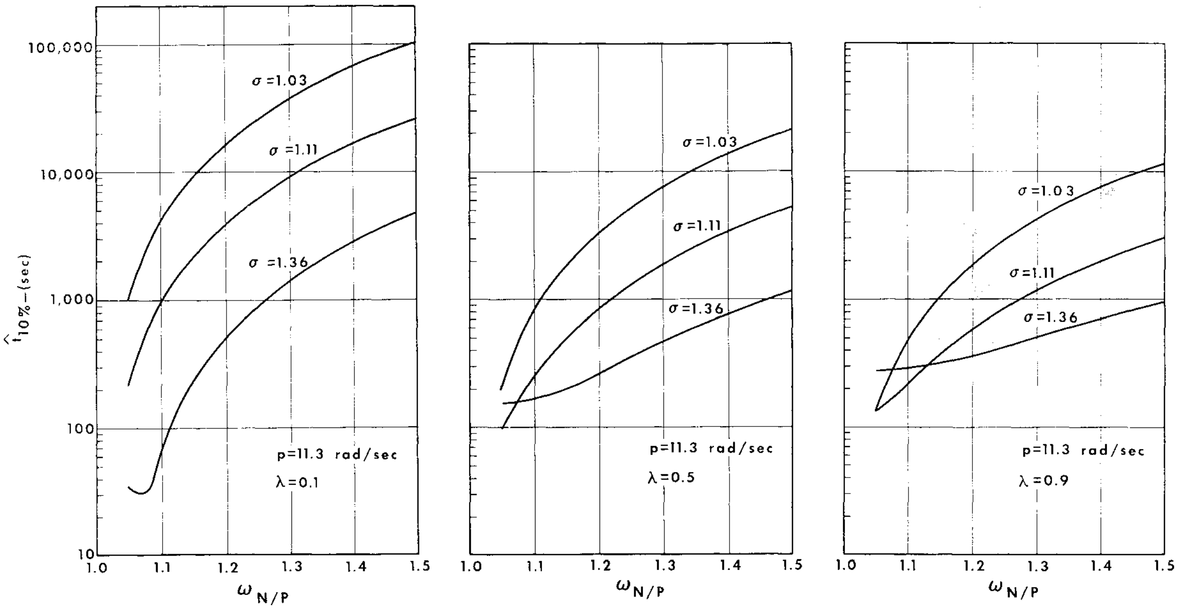

Fig. 4 Computer solution for damping times: $ma^2/I_y = 6.46 \times 10^{-4}$; $p = 11.3$ rad/sec.

can now be seen and, in effect, the damping factor F can be seen to be the real part of the factors of r^* in the brackets since this will be the part in phase with the velocity. After the substitution $i\Omega$ for s and considerable algebraic manipulation, the approximate formula for $F/2$ results. Results obtained using the approximate formula compare favorably with those obtained using the digital computer program.

In order to estimate the maximum excursion of y, the following approximate formula is used:

$$y_{\max} \approx r_0 a(p + \Omega)/2\lambda\omega_n\Omega$$

This formula also is derived on the assumption that the sinusoidal character of Ω is not appreciably changed by the presence of the damper.

Angular Displacement of Spin Axis

The foregoing discussion pertained to the convergence of lateral angular rates only with the intuitive result that the angular displacements must also be converging. This convergence can be demonstrated by further analysis and tests.

The general character of the motion can be determined with reasonable accuracy for small angular displacements by the linearized Euler angular rate equations for constant spin rate as given in Ref. 2:

$$q = r_0 \exp(-Ft/2) \sin\Omega t$$

$$r = r_0 \exp(-Ft/2) \cos\Omega t$$

and

$$\dot{\theta} = q \cos\phi - r \sin\phi$$

$$\dot{\psi} = r \cos\phi + q \sin\phi$$

$$\dot{\phi} \cong p$$

$$\phi \cong pt$$

Ignoring initial conditions and integrating the rate equations for $\dot{\theta}$ and $\dot{\psi}$ in closed form gives the following expression for the spin-axis angular displacement:

$$\delta = \theta + i\psi = [r_0/(p + \Omega)] \times \{\exp[-Ft/2 + i(p + \Omega)t] - 1\}$$

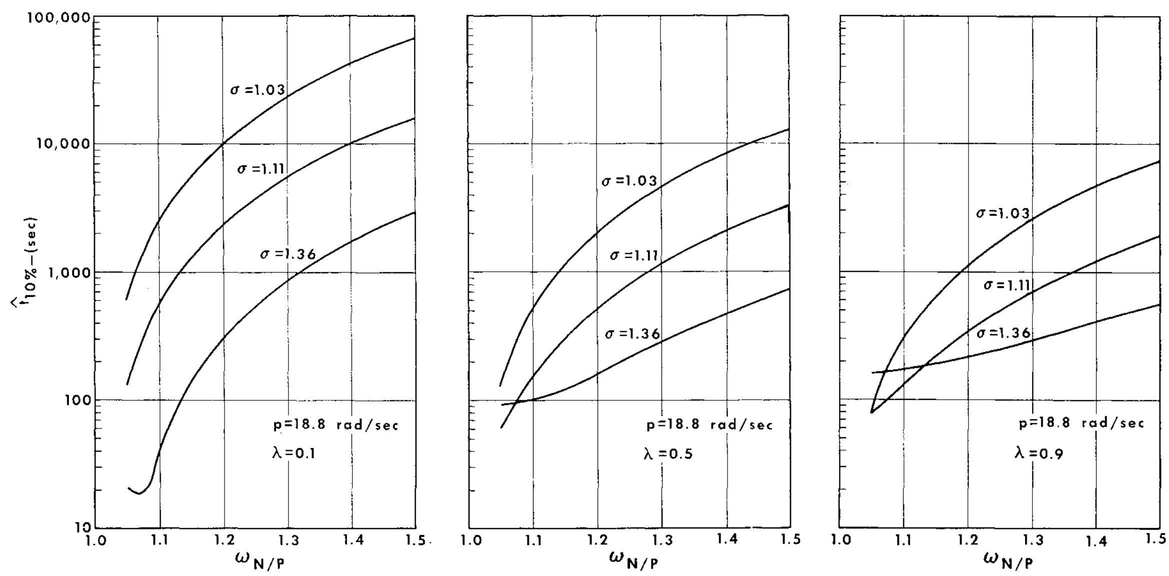

Fig. 5 Computer solution for damping times: $ma^2/I_y = 6.46 \times 10^{-4}$; $p = 18.8$ rad/sec.

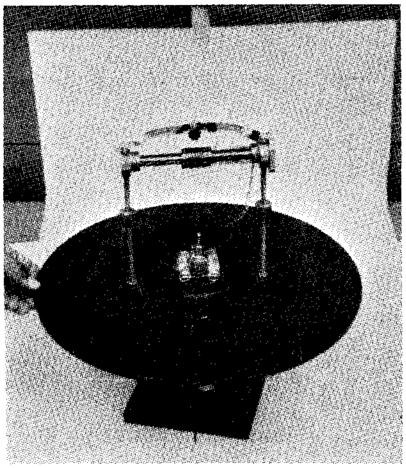

Fig. 6 Test setup.

where the angle δ is written in complex form for convenience, and certain insignificant terms from the integration have been omitted. This expression indicates that the spinning vehicle will oscillate about and eventually converge to a displacement of approximately $r_0/(p + \Omega)$ from its original inertial space position at time zero.

Test Program

A small spin table was used as a testing device to verify the principle of the mass-spring damper. The table is shown in Fig. 6 with damper installed. Adjustable balance weights were installed on the underside of the table so that the table was balanced about a small hemispherical bearing in the center of the table. This approaches the desired zero-g condition since it allows 30° or so of lateral rotational motion and spin motion when the table is supported on the spindle and pedestal arrangement. Balancing the table with or without the damper tends to eliminate gravity moments, thus approaching a zero-g system, since the c.g. of the system is then at the support bearing.

The damper was a ball bearing sliding on a lubricated rod and connected through a spring to a housing at one end of the rod. Calibration of the spring mass mechanism was accomplished by using a vibrator. Sinusoidal inputs were applied and the damping was adjusted by using silicone grease. The resulting damped fundamental frequency of the damper was approximately 3 cps with $\lambda \approx 0.5$.

Figure 6 shows the device with the damper mounted for testing. A small light fixed above the centroid was used to obtain pictures of the motion of the axis of symmetry. This light was located in the center of the arched member at the top of the device. A hand drill with a special drive wheel was built and calibrated to spin the table to 3 cps. Time-exposure pictures were taken in a darkened room showing the time history of the position of the axis of symmetry as the table was spun, an impulse applied, and the motion damped.

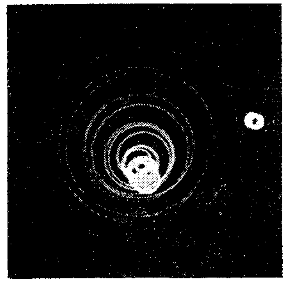

Fig. 7 Test with damper: 5-sec exposure time, impulse applied at 1 sec.

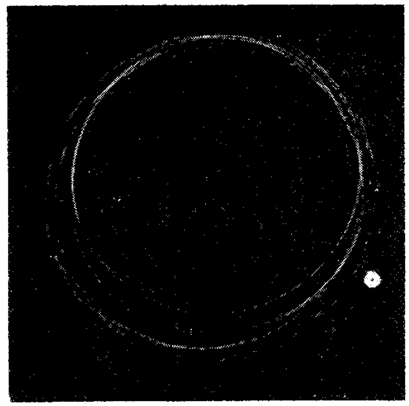

Fig. 8 Test without damper: 5-sec exposure time, impulse applied at 1 sec.

Figures 7 and 8 show some comparative results for the tests with and without the damper. The time for the light beam (or spin axis) to traverse a complete circle is about $\frac{1}{3}$ sec.

In approximately 6 sec, all of the coming motion was damped. The ratio ma^2/I for this test setup was 1.35×10^{-2}, and ω_n/p was ≈ 1.1.

Conclusions

Analysis and laboratory tests have demonstrated the feasibility of a passive damper of the configuration shown in Fig. 6. Design curves for a particular ratio of ma^2/I are presented, and the approximate formula indicates damping times are directly proportional to this ratio, all other factors remaining constant. Thus, damping times for other ratios of ma^2/I also can be approximately determined from the curves.

References

[1] Gillespie, W., Jr., Eide, D. G., and Churgin, A. B., "Some notes on attitude control of earth satellite vehicles," NASA TN D-40 (1959).

[2] Suddath, J. H., "A theoretical study of the angular motions of spinning bodies in space," NASA TR R-83 (1961).

[3] Goldstein, H., *Classical Mechanics* (Addison-Wesley Publishing Co., Reading, Mass., 1959), Chap. V, p. 158; Chap. I, p. 21.

[4] Levy, E. C., *A New Table of Laplace Transformation Pairs* (Los Angeles, Calif., 1959), Chap. III, p. 29.